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MODELING DAMPED MASS-SPRING SYSTEM IN MATLAB SIMULINK ®

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Abstract:-
The vibrations have turn out to be all the time more significant in recent years owing to the present trend toward higher-speed machines and lighter structures. Most mechanical and structural systems have distributed mass, elasticity, and damping. Ordinary differential equations (ODEs) play a vital role in such mechanical and structural systems. In this paper we construct a Mathematical model and Simulink Model for the damped mass-spring system by using second law of motion to the masses with the forces acting by the spring and force by any external sources. Simulink Model developed by using block diagram from the different libraries of Simulink.

Key Words: Mass Spring System, Over Damping, Critical Damping, Under Damping, Simulink Model

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1. Introduction

Ideally, one could imagine that vibrating systems are free of damping. However, in actual fact, all vibrations are damped to some degree by resistance forces. The important applications in which the mass-spring damper model appears are vibration control [1] , estimation of contact parameters [2], control in robotics [3].Damped forces can be caused by dry friction, or Coulomb friction, or by inside. A damping is type of the opposite of a spring, with the exception of it operates on relative rate of change of displacement rather than displacement.

Mass Spring systems are simple mechanical models that allow us to build a simple Mathematical Model using dynamics. We work on spring that connected with mass. By the Newton's second law, we know that $m\frac{dx''(t)}{dt}=F$ [6, 7, 8, 10] Substitute spring force relation we get $m\frac{dx''(t)}{dt}=-kx$. Every simple harmonic oscillation can be written in this same basic form. Springs frequently occur actually as a metal-coil, and their idealizations have simple behaviour, compressed-spring will result by spring pushing, and stretched-spring will required to pull back to the initial position, so any displacement along the axis of the spring will be countered by an opposite force that will tend to move the spring back to its original position. When there is mass-spring system with a damping force, this force is generally proportional to the velocity and is always opposing the applied force [7] $F_d=-c\frac{dx}{dt}$ the c is called damping constant. The model equation is a second order ODE based linear and homogeneous model can be simulated by using Simulink [8]. Simulink built on top of MATLAB, It arrange for a graphical user interface that uses different kinds of origins called blocks to create a simulation Model [4] of a dynamic system; that is, a system that can be model by differential equations whose independent variable is time.

Problem Formulation

A spring-mass composition consists of mass-points that are connected by springs. The spring dynamics (including damping) is responsible for the generation of the internal forces considering both spring stiffness and spring damping. The point dynamics [4] determines the acceleration velocity and the new position of a mass point taking into account the internal and external forces. Let $x(t)$ represents the displacement of the body from its equilibrium point at time t, measured downward and f is the sum of forces acting on mass. Then Newton's second Law $m\frac{dx''(t)}{dt}=f(t)$ determining f , there are four distinct forces to be considered:

(i) Gravitational force (downward force),
(ii) Spring force (up or down force),
(iii) Damping force (up or down),
(iv) External force (up or down).

Neglecting Gravitational force, we consider the other forces.

2.1 Spring Force

The spring force F_S obey Hook's law there are two cases for the displacement

(i) When $x > 0$ then spring is stretched and the spring force acts upward. That is

$F_s = -ks_o - kx = -k(s_o + x)$

(ii) When $x < 0$ the spring is compressed. In this way spring force acts downward. therefore

$F_s = -ks_o + k\,|\,x\,| = -ks_o + k(-x) = -ks_o - kx = -k(s_o + x)$

In both cases we have $F_s = -k(s_o + x)$

2.2 Damping Force

The damped force or resistive force [5, 7] F_d acts in opposite path as motion of mass. The damping force F_d may probably be due to air resistance, inner energy dissipation, due to performance of spring, friction, or a mechanical device resistive force to mass and is proportional to the rate at which the displacement change.

In particular the rate of change of displacement follows two ways, we have

(i) When $x' > 0$, then displacement is increasing and mass is moving downward. Thus the action of damping force is upward and hence

$F_d = -c\dfrac{dx}{dt}$, where c> 0.

(ii) When $x' < 0$ then displacement is decreasing and mass is going towards upward. Thus damping force acts downward and hence

$F_d = +c\left(-\dfrac{dx}{dt}\right) = -c\dfrac{dx}{dt}$,where c > 0.

In both these cases,

$$F_d(t) = -c\frac{dx(t)}{dt}, \quad c > 0$$

2.3 Damped Mass-Spring Systems

The equations from sections 2.1 and 2.2 are combining by using Newton's second Law of Motion in the forma second-order homogeneous linear differential equation [5] for dependent variable displacement as a function of independent variable time.

$$m\frac{d^2x}{dt^2} = F(t) + mg + F_s(t) + F_d(t)$$
$$= F(t) + -k[s_o + x(t)] - c\,x'(t) + mg$$

If $F(t) = 0$ then the characteristics equation is $m\dfrac{d^2x}{dt^2} + c\dfrac{dx}{dt} + kx = 0$, and then the roots are

$$\lambda = \frac{-C}{2m} \pm \frac{1}{2m}\sqrt{c^2 - 4mk}$$

The damped model has three choices for oscillations:

Over-damped: when c^2 is greater than 4mk;

Critically damped: when c^2 is equal to 4mk;
Under-damped: when c^2 is less than 4mk

2.3 Simulink Model for Case 1:

Let us Consider a system where, $mass = 1$; $C = 3, k = 1$, as $c^2 > 4mk$ then it is over-damped system.

The model equation is $\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + x = 0$, with the conditions

$$x(0) = 0; \; x'(0) = 1.0$$

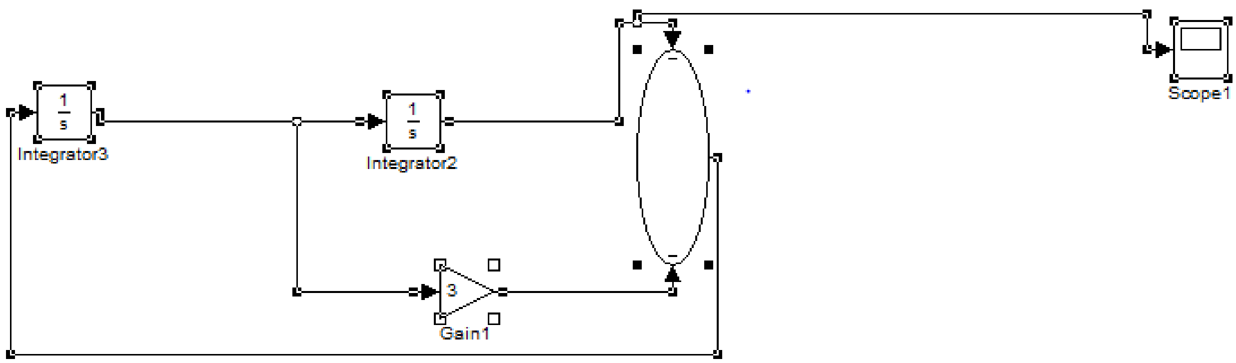

Figure 1: Simulink model for over damping

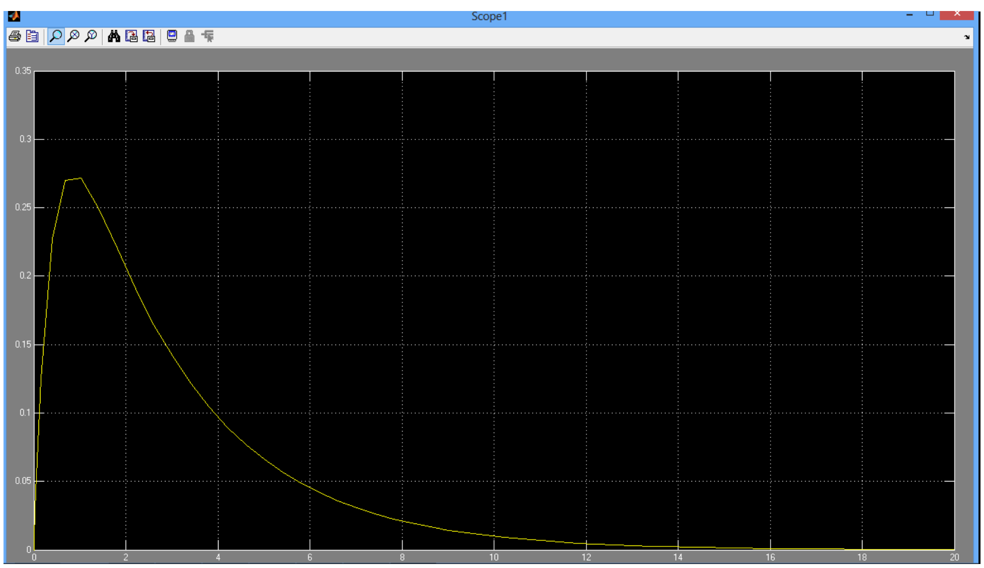

Figure 2: Scop of Simulink model for over damping

It is shown from the Scope1 that there is no oscillations, The mass displacement first increases and goes to a peak value and then starts coming back to its original zero position in exponential decay manner.

2.4 Simulink Model for Case 2:

Let us Consider a system where, $mass = 1$; $C = 2$, $k = 1$. Then, $c^2 = 4mk$ and it is a critically – damped system.The model equation is $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + x = 0$ with conditions:

$$x(0) = 0;\ x'(0) = 1.0$$

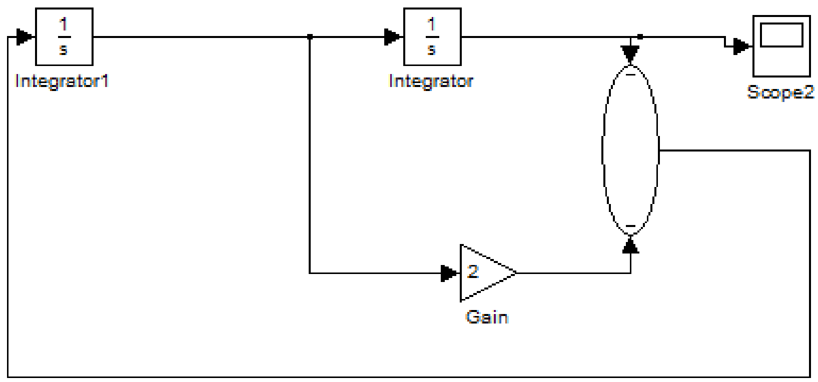

Figure 3: Simulink model for critically – damping

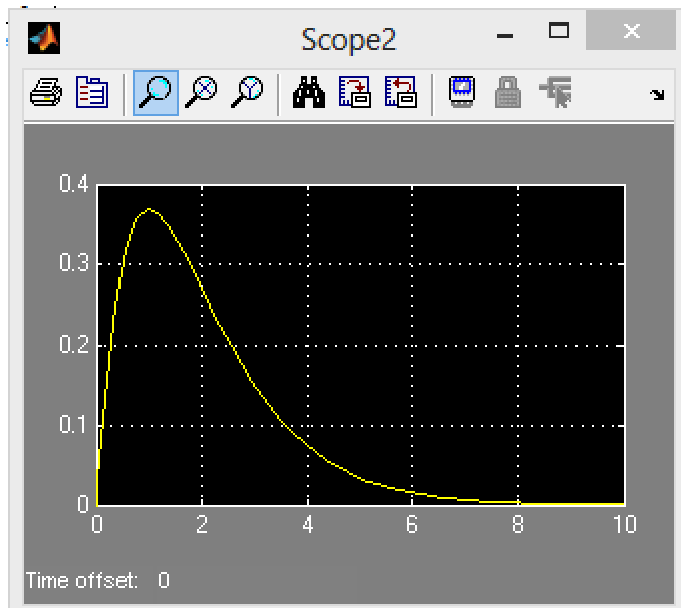

Figure 4: Scope of Simulink model for critically – damping

Also it is shown from the Scope2 that there is no oscillations the mass displacement first increases and goes to a peak value and then starts coming back to its original zero position in exponential decay manner.

Simulink Model for Case 3:

Let us Consider a system where, $mass = 1$; $C = 1$, $k = 1$. As ,$C^2 < 4mk$ then it is a under-damped system.

The model equation is $\frac{d^2x}{dt^2}+\frac{dx}{dt}+x=0$ with initial conditions are $x(0) = 0; x'(0) = 1.0$

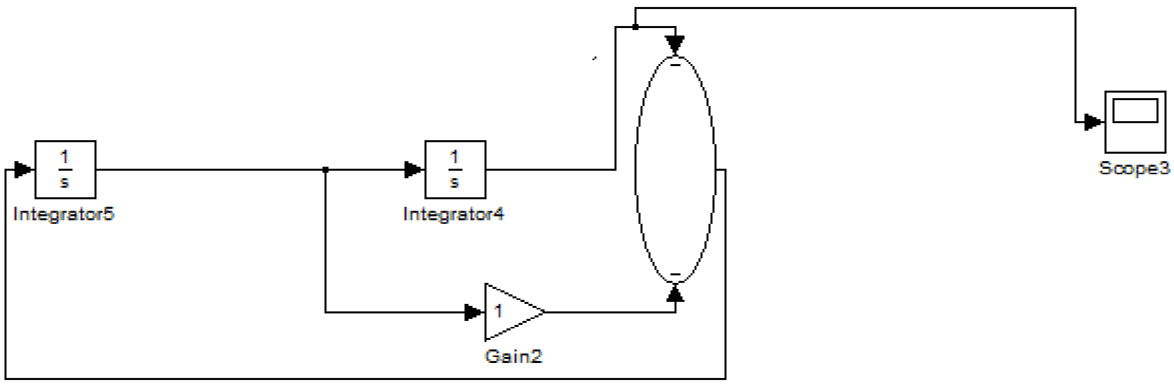

Fig 5: Simulink model for under – damping

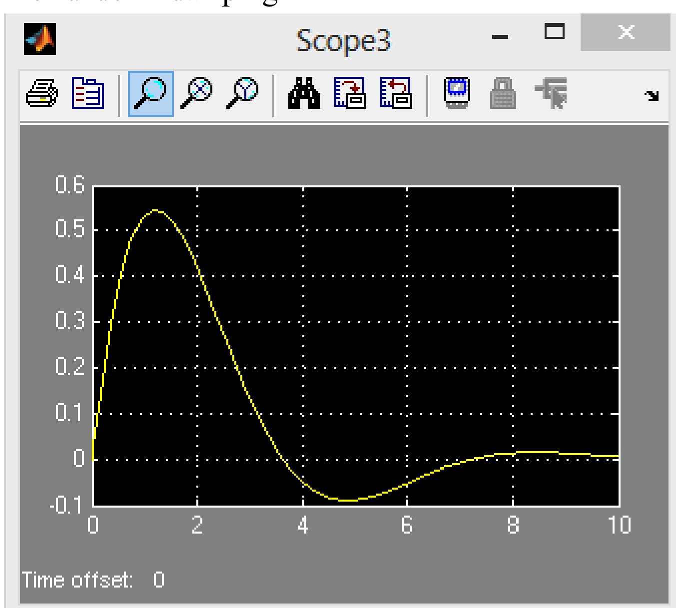

Fig 6: Scope of Simulink model for under– damping

This model shows oscillations. The mass displacements first increases and goes to a peak value and then start oscillating with decreasing amplitudes.

Comparison of all three cases;

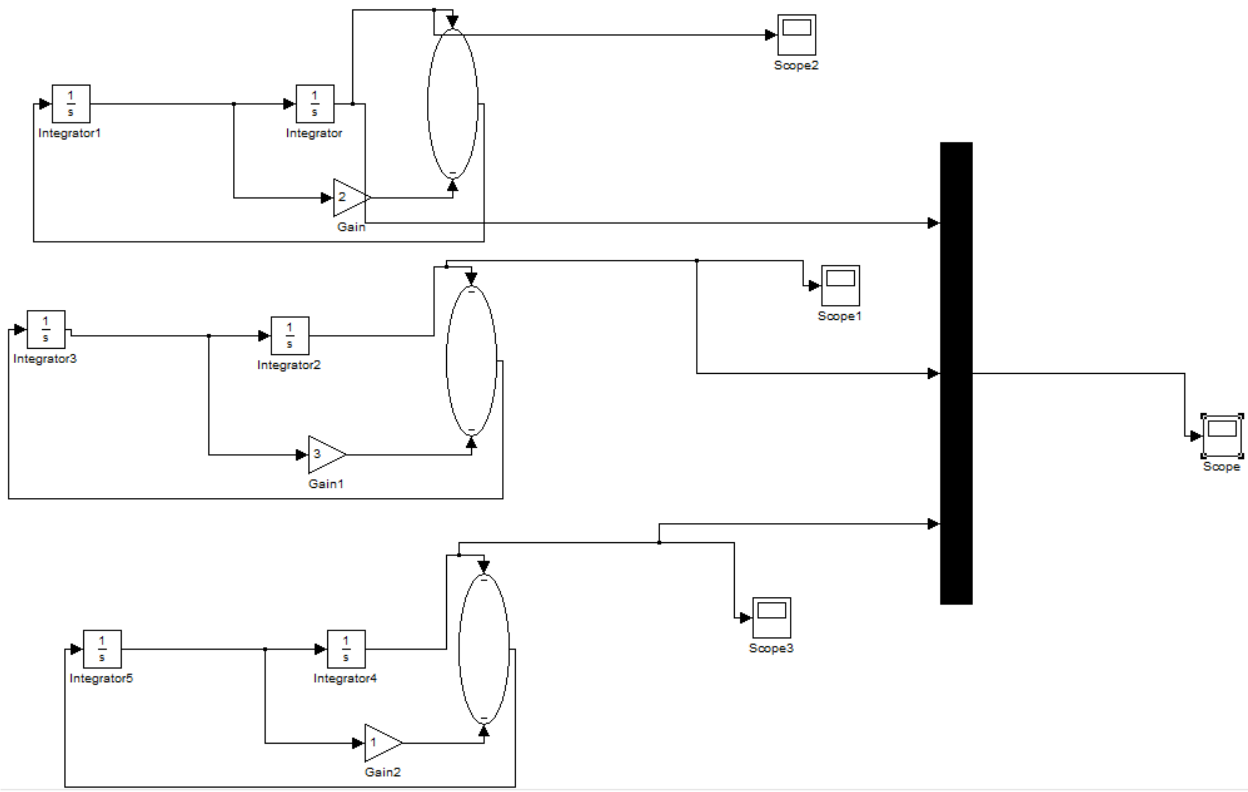

Figure 7: Simulink model for over ,critical and under damping mass spring system

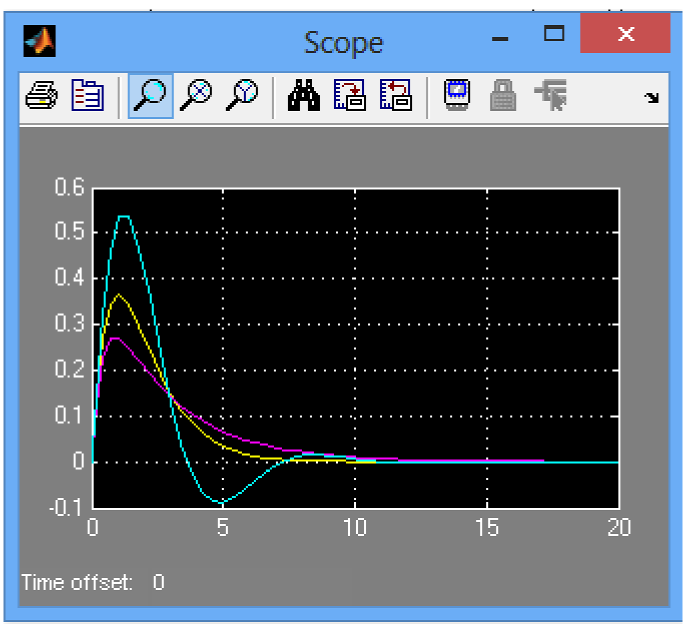

Figure 8: Scope for Simulink model of over, critical and under damping mass spring system

3. Conclusion

In this paper we investigate mathematical modelling of damped Mass spring system in Matlab /Simulink. The Scope1 and scope 2 indicate no oscillations, the mass displacement first increases and then starts coming back to its original zero position in exponential decay manner. As the solution of such second order differential equations provides exponential type functions and our Simulink Model 1 and Model 2 represent the same behavior. But in scope 3 we observe that there is some oscillation due to the over damping. As the solution of such differential equation gives a function which generates sine and cosine waves, the same behavior appear in our Simulink Model 3.It is also find that Simulink Models provide quick result of second order homogeneous differential equation based model as compared to analytical and numerical techniques.

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